Hyperuniform Point Configurations

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Hyperuniform point configurations in d-dimensional Euclidean space are characterized by an unusual suppression of density fluctuations at large lengths scales. All perfect crystals, perfect quasicrystals and exotic disordered patterns are hyperuniform. Thus, the hyperuniformity concept provides a unified theoretical framework to categorize a large class of ordered and disordered systems. Disordered hyperuniform many-particle systems can be regarded to be new states of disordered matter in that they behave more like crystals or quasicrystals in the manner in which they suppress large-scale density fluctuations, and yet are also like liquids and glasses because they are statistically isotropic structures with no Bragg peaks. I will provide an overview of the hyperuniformity concept and its generalizations. Topics covered include how optimal hyperuniform point configurations can be posed as energy-minimization problems; rank order of crystals, quasicrystals and disordered hyperuniform systems via a hyperuniformity index; classical ground states that are disordered, hyperuniform and highly degenerate; determinantal point processes; zeros of the Riemann zeta function, and directional hyperuniformity. I will also discuss how the hyperuniformity concept motivated a recent study that has uncovered previously unknown multiscale order in the prime numbers.